

Analysis of polarized diffraction images of human red blood cells: a numerical study: supplement

WENJIN WANG,^{1,2,*} LI MIN,^{1,2} PENG TIAN,^{1,2}  CHAO WU,^{1,3} JING LIU,^{1,4} AND XIN-HUA HU^{1,5} 

¹*Institute for Advanced Optics, Hunan Institute of Science and Technology, Yueyang, Hunan 414006, China*

²*School of Physics & Electronics Science and Technology, Hunan Institute of Science and Technology, Yueyang, Hunan 414006, China*

³*Intelligent Manufacturing Research Institute, South-Central University for Nationalities, Wuhan, Hubei 430074, China*

⁴*School of Information Science and Engineering, Hunan Institute of Science and Technology, Yueyang, Hunan 414006, China*

⁵*Department of Physics, East Carolina University, Greenville, NC 27858, USA*

*wangwenjin1987@126.com

This supplement published with Optica Publishing Group on 3 February 2022 by The Authors under the terms of the [Creative Commons Attribution 4.0 License](https://creativecommons.org/licenses/by/4.0/) in the format provided by the authors and unedited. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

Supplement DOI: <https://doi.org/10.6084/m9.figshare.19087004>

Parent Article DOI: <https://doi.org/10.1364/BOE.445370>

Gray Level Co-occurrence Matrix (GLCM) for Diffraction Image Analysis

According to the introduction of Haralick, Gray Level Co-occurrence Matrix (GLCM) algorithm summarized as mathematical and statistical texture analysis processes performed to extract important second order statistical texture features from monochromatic images. These images represented two-dimensional pixels array, and each pixel contains a quantized grey level. The co-occurrence frequency defined for two neighboring pixels that are separated by a displacement vector. The number of the pixels having the same displacement vector (based on the distance and angle) represented as an element in gray level co-occurrence matrix.

In other words, the gray tone of a rectangular Image I with N_x horizontal resolution pixels, and N_y vertical resolution pixels is quantized by N_g levels. $L_x = \{1, 2, \dots, N_x\}$ and $L_y = \{1, 2, \dots, N_y\}$, are the horizontal and the vertical spatial domains respectively, and $G = \{1, 2, \dots, N_g\}$ is the quantized gray level tone. The set $L_x \times L_y$ represent the set of pixels of the image sorted by their row-column labels. An input image I can be regarded as a function that assigns some gray level in G to each pixel in $L_x \times L_y$. It is assumed that the texture information in an image I is contained in the overall or average spatial relationship. Let's denote $p(i, j, d)$ as the "co-occurrence" frequency of two neighboring pixels that are separated by the displacement vector $\mathbf{d} = (d, \theta)$ with one pixel intensity of gray level i and the other of gray level j . d is the offset separation distance between the two pixels and θ is the specified angular direction, usually ($\theta = 0^\circ, 45^\circ, 90^\circ, \text{ and } 135^\circ$) [1].

A GLCM matrix P can be obtained with the elements as frequency $p(i, j, d)$. It is easy to show that the matrix is symmetric since $p(i, j, d) = p(j, i, d)$ and depends on the choice of d . For example, suppose an image having 4×4 pixels with gray level range from 0 to 3 as shown below^[1]:

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

The frequencies of the GLCM can be found for the horizontal direction ($\theta = 0$) and $d = 1$:

$P_{0^\circ} =$	4	2	1	0
	2	4	0	0
	1	0	6	1
	0	0	1	2

For ($\theta = 45^\circ$) and $d = 1$ is:

$$P_{45^\circ} =$$

4	1	0	0
1	2	2	0
0	2	4	1
0	0	1	0

For $(\theta = 90^\circ)$ and $d = 1$ is:

$$P_{90^\circ} =$$

6	0	2	0
0	4	2	0
2	2	2	2
0	0	2	0

For $(\theta = 135^\circ)$ and $d = 1$ is:

$$P_{135^\circ} =$$

4	2	1	0
2	4	0	0
1	0	6	1
0	0	1	2

After computing the frequencies of all possible gray level pairs, the GLCM usually normalized to the total number of neighboring pixels pair for the calculated matrix $P^{[1]}$.

The following table shows the definitions for most relevant texture feature of GLCM used in image analysis:

	Parameter	Abbreviation	Mathematical definition
1	Angular Second Moment (or energy or homogeneity)	ASM	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \{p(i, j)\}^2$
2	Contrast (Inertia)	CON	$\sum_{k=0}^{G-1} k^2 p_{x-y}(k)$
3	Correlation	COR	$\frac{\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (ij) p(i, j) - \mu_x \mu_y}{\sigma_x \sigma_y}$
4	Variance (Sum of Squares)	VAR	$\sum_{i=0}^{G-1} (i - \mu_x)^2 p_x(i)$
5	Inverse Difference Moment (Local Homogeneity)	IDM	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{1}{1 + (i - j)^2} p(i, j)$
6	Sum Average	SAV	$\sum_{k=0}^{2G-2} k p_{x+y}(k)$
7	Sum Entropy	SEN	$-\sum_{k=0}^{2G-2} p_{x+y}(k) \cdot \log(p_{x+y}(k))$
8	Sum Variance	SVA	$\sum_{k=0}^{2G-2} (k - SEN)^2 p_{x+y}(k)$

9	Entropy	ENT	$-\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} p(i, j) \cdot \log(p(i, j))$
10	Difference Entropy	DEN	$-\sum_{k=0}^{G-1} p_{x-y}(k) \cdot \log(p_{x-y}(k))$
11	Difference Variance	DVA	$CON - \left(\sum_{k=0}^{G-1} k p_{x-y}(k) \right)^2$
12	Dissimilarity	DIS	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} i - j p(i, j)$
13	Cluster shade	CLS	$= \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i + j - 2\mu_x)^3 p(i, j)$
14	Cluster Prominence	CLP	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i + j - 2\mu_x)^4 p(i, j)$
15	Maximum probability	MAP	$\max(p(i, j))$